Section 8: Getting Things into Space: Rockets and Launch Requirements

To place an object in orbit, a rocket must be able to do two things: carry the object to the proper altitude and give it the correct speed at that altitude. Even a short-range missile can launch a payload to altitudes of several hundred kilometers (at which point it will fall back to Earth), whereas placing an object into low earth orbit requires a much more powerful rocket.[1] Developing rockets powerful enough to place satellites in orbit is a difficult technical challenge; currently, only a handful of countries have developed this capability.

USING MISSILES TO LAUNCH PAYLOADS TO HIGH ALTITUDES: THE "1/2 RULE"

A useful rule of thumb is that a ballistic missile that can launch a given payload to a maximum range *R* on the Earth can launch that same payload vertically to an altitude of roughly *R*/2. This relation is exact in the case of a flat Earth and therefore holds for missiles with ranges up to a couple thousand kilometers (the Earth appears essentially flat over those distances, which are small compared to the radius of the Earth). But the rule continues to hold approximately for even intercontinental range missiles (see Appendix B to Section 8).

Changing the payload of the missile changes both its maximum range and its maximum altitude, but these two distances continue to be related by the 1/2 Rule. For example, a Scud missile, which has a maximum range of 300 km with a one-ton payload, would be able to launch a one-ton payload vertically to an altitude of 150 km. By reducing the payload, the Scud missile could launch it to higher altitudes. Reducing the payload of a Scud missile by one-half, to 500 kg, would give it a maximum range of about 440 km or allow it to launch the payload to a maximum altitude of about 220 km. Reducing the payload to 250 kg would increase the maximum range to about 560 km, and the maximum altitude to about 280 km.

PUTTING OBJECTS INTO ORBIT

Recall from Section 4 that a satellite in low earth orbit has a speed of 7 to 8 km/s. The rocket placing the satellite into orbit must therefore be able to reach that speed.

1. For example, the potential energy of a mass lifted to a 300 km altitude is less than 3% of the kinetic energy of the same mass in a circular orbit at that altitude. (See Appendix A to Section 8 for a discussion of the potential and kinetic energies of orbits.)

It is useful to compare this speed to that of ballistic missiles of various ranges.[2] A short-range missile, such as the 300-km range Scud missile that the Soviet Union developed in the 1960s, reaches a top speed of about 1.4 km/s. For a missile to reach a range of 1,000 km, it must be able to reach a speed of 3 km/s. An intercontinental ballistic missile, similar to those the United States and Russia deploy as part of their strategic nuclear forces, is capable of reaching a quarter of the way around the Earth (10,000–12,000 km) and reaches a speed in excess of 7 km/s.

The similarity in speed between an intercontinental missile and a rocket needed for space-launch is the reason that similar technology can be used for both and that countries have generally developed the two capabilities at the same time.

Note, however, that even an intercontinental-range missile cannot place its full payload into orbit. A 10,000-km range missile typically burns out at an altitude of several hundred kilometers. Its speed of roughly 7 km/s is about 10% too low to place a satellite in a circular orbit at that altitude. Reaching the necessary speed would require reducing the payload by roughly a third.[3] This fact is important in comparing space-based and missile-launched weapons, as is done in Section 9.

Placing an object into orbit is thus technically demanding. It is also expensive. A rough rule of thumb is that a modern rocket can deliver into orbit a payload that is only a few percent of the rocket's overall mass. Since the size of the rocket needed to put a satellite into orbit scales with the mass of the satellite, there is a tremendous incentive to keep the mass of satellites as low as possible.

Table 8.1 gives data on several space-launch vehicles, including the lift-off mass of the launcher and the mass that it can place in three different types of orbits: circular low earth orbits with altitudes of a few hundred kilometers; geosynchronous transfer orbits, which are elliptical orbits with perigee typically at a few hundred kilometers and apogee at geosynchronous altitude of approximately 36,000 km; and sun-synchronous orbits, which typically have altitudes below 1,000 km and an inclination near 90°. The table shows that modern launchers have lift-off masses of 200 to 700 metric tons and are able to place 2.5% to 4% of their lift-off mass in low earth orbit. As the table also shows, for a given launcher, the mass that can be placed in a geosynchronous

2. A ballistic missile warhead is not powered throughout its flight. Instead the missile rapidly accelerates the warhead to high speed and then releases it, so that for most of its trajectory the warhead is falling through space (this free-falling motion is called ballistic, accounting for the term ballistic missile). The range that a warhead can reach depends on how fast the rocket booster is traveling when it releases the warhead, just as the distance a baseball travels depends on how fast it is moving when it leaves your hand. The rocket booster reaches it maximum speed at "burnout," when the rocket finishes burning its fuel.

3. Steve Fetter, University of Maryland, personal communication, July 2004. For several different missiles, Fetter calculated the mass that the missile could launch into low earth orbit (200 and 500 km altitude) and compared it with the mass he calculated the same missile could send to 10,000 km range.

transfer orbit is roughly half the mass that can be placed in low earth orbit; this is comparable to the mass that can be placed in a sun-synchronous orbit.

Table 8.1: This table shows the satellite mass that various space launchers can place in different orbits.[4] M_{LEO}, M_{GTO}, and M_{SSO} are the masses that can be placed in low earth orbit (LEO), geosynchronous transfer orbit (GTO), and sun-synchronous orbit (SSO), respectively. (A metric ton is 1,000 kg.) In the second column, the numbers in parentheses are the altitude h and inclination θ of the orbits. In the third column, these numbers are the perigee of the elliptical orbit, and the inclination θ. In all cases the apogee is at geosynchronous altitude of roughly 36,000 km. The fifth column gives the lift-off mass of the launcher, M_{LO}. The final three columns give the ratio of the satellite mass to the lift-off mass for the three types of orbits.

Launcher	M_{LEO} (metric tons) (h, θ)	M_{GTO} (metric tons) (perigee, θ)	M_{SSO} (metric tons)	M_{LO} (metric tons)	$\frac{M_{LEO}}{M_{LO}}$	$\frac{M_{GTO}}{M_{LO}}$	$\frac{M_{SSO}}{M_{LO}}$
Ariane 4 (AR44L) (Europe)	10.2 (200km, 5.2°) 8.2 (200km, 90°)	 4.8 (185km, 7°)	 6.5	 470	2.6% 1.7%	 1.0%	 1.4%
Ariane 5 (Europe)	18 (550km, 28.5°)	6.8 (580km, 7°)	12	737	2.4%	0.92%	1.6%
Atlas IIA (USA)	7.3 (185km, 28.5°) 6.2 (185km, 90°)	 3.1 (167km, 27°)		 188	3.9% 3.3%	 1.6%	
Atlas V 550 (USA)	20 (185km, 28.5°) 17 (185km, 90°)	 8.2 (167km, 27°)		 540	3.7% 3.1%	 1.5%	
Delta III (USA)	8.3 (185km, 28.7°) 6.8 (200km, 90°)	 8.3 (200km, 28.7°)	 6.1	 302	2.8% 2.2%	 1.3%	 2.0%
GSLV (India)	5 (200km, 45°)	2.5 (185km, 18°)	2.2	402	1.2%	0.62%	0.55%
H-2 (Japan)	10.6 (200km, 30.4°)	3.9 (250km, 28.5°)	4.2	260	3.9%	1.5%	1.6%
H-2 A2024 (Japan)	11.7 (300km, 30.4°)	5.0 (250km, 28.5°)	5.3	289	4.1%	1.7%	1.8%
LM-3B (China)	11.2 (200km, 28.5°)	5.1 (180km, 28.5°)	6.0	426	2.6%	1.2%	1.4%
Proton K (Russia)	19.8 (186km, 51.6°)	4.9 (4200km, 23.4°)	3.6	692	2.9%	0.71%	0.52%
PSLV (India)	3.7 (200km, 49.5°)	0.8 (185km, 18°)	1.3	294	1.3%	0.3%	0.4%

4. S.J. Isakowitz, J.P. Hopkins, Jr, and J.B. Hopkins, *International Reference Guide to Space Launch Systems*, 3rd ed. (Reston, VA: American Institute of Aeronautics and Astronautics, 1999).

Factors That Affect Launch Capability

The capability of a given launcher to place objects in orbit depends on many factors, as listed below.

- *The mass being lifted into space (the payload).* As a consequence of the rocket equation, the propellant in the missile cannot accelerate a massive payload to as high a velocity, and thus lift it to as high an altitude, as it can a lower mass payload. Moreover, the heavier the payload, the more gravity slows the rocket as it travels to high altitudes. As a result, the more massive the payload, the lower the altitude at which the rocket can place this mass in orbit.
- *The location of the launch site and the direction of the launch.* The rotation of the Earth gives a rocket an eastward velocity even before it is launched.[5] If the rocket is launched to the east, it can use this velocity to increase its speed. Since the speed of the Earth's surface is greatest at the equator (0.456 km/s), launching from a location at low latitudes (near the equator) increases the rocket's speed and therefore increases its launch capability. In addition, for launches into geostationary orbit, launching from near the equator can place the satellite into an orbital plane with the correct inclination; launching from higher latitudes requires the launcher to use propellant to rotate the orbital plane to make it equatorial.

  For example, a rocket launched from the French Kourou launch site at 5.23° latitude could carry 20% more mass into a geosynchronous transfer orbit than could the same rocket from the Kazakh Baikonur launch site at 46° latitude.[6] For a launch site at 70° latitude, the rocket could only carry half as much mass as one launched from Kourou.

  Similarly, if the rocket is not able to launch eastward, it cannot take full advantage of the speed of the Earth's rotation, and this reduces its launch capability. This can happen, for example, if the satellite is being launched into a polar orbit, in which case the rocket is launched toward the north or south. Or the launch directions may be restricted so that the rocket does not fly over populated areas early in flight. This constraint may impose a fuel-costly orbital maneuver to reach the desired final orbit, and thus reduce the launch capability. Both India and Israel are in this situation. The rel-

5. The additional speed that a launcher can use from the Earth's rotation is given by: $V = 0.456 \cos\theta \cos\alpha$, where θ is the latitude of the launch site and α is the angle between the direction of the launch and due east.

6. The launch from Baikonur would lose 0.14 km/s from the Earth's rotation relative to a launch from Kourou, and rotating the orbital plane from an inclination of 46° to 0° would require ΔV = 2.4 km/s, assuming it was done once the satellite was in geostationary orbit.

atively low mass ratios shown in the last three columns of Table 8.1 for the Indian GSLV and PSLV rockets reflect both India's somewhat less mature technology and the geographic restrictions on the directions it can launch, which prevent it from taking full advantage of the rotation of the Earth.[7]

Another example is the North Korean attempt to launch a satellite in August 1998, which was launched eastward over Japan. Because the rocket passed over Japan, many saw this act as threatening; however, this trajectory was likely chosen to take maximum advantage of the Earth's rotational speed.

- *Details of the orbit.* The altitude, shape, and inclination of the orbit all affect orbital launch capability. For example, if V_h is the speed a satellite needs to be placed in a circular orbit at altitude h, a higher speed equal to $\sqrt{1+e}\; V_h$ is required for a satellite to be placed into an elliptical orbit (with eccentricity e) at its perigee point at an altitude h.

  As noted above, if the satellite needs to be placed in a highly inclined orbit, it is likely to be launched in a direction that does not allow it to make maximum use of the Earth's rotational speed. This can be illustrated by comparing the masses that can be launched into low earth orbits with different inclinations in Table 8.1. Four of the entries provide data for launches into polar orbits and into orbits with lower inclinations with the same or comparable altitude: Ariane 4, Atlas IIA, Atlas V 550, and Delta III. In each case, the rocket is able to place 20% more mass into the low inclination orbit than the polar orbit.

  Alternately, since a satellite cannot be launched into an orbit with inclination less than the latitude of the launch site (see Section 4), if a satellite is launched from a location in the mid-latitudes but is intended for an orbit with inclination near zero, the satellite must maneuver to change orbital planes, which requires additional fuel mass to be carried into orbit, which reduces the launch capability.

Placing Satellites in Geostationary Orbit

Satellites are typically placed in geostationary orbits in two steps. The first step is to launch the satellite into a parking orbit, which is typically at low altitude (200 to 300 km). The second step is to maneuver the satellite into an elliptical Hohmann transfer orbit, or geosynchronous transfer orbit (GTO), to change the orbit from low earth orbit to geosynchronous orbit (see Figure 6.2). The transfer orbit has its perigee at the parking orbit's altitude and its apogee at geosynchronous altitude. A ΔV of 2.4 km/s is required to place the

7. Isakowitz et al., 310.

satellite into GTO from the parking orbit, and another ΔV of 1.5 km/s is required to circularize the orbit at GEO, for a total ΔV of 3.9 km/s for reaching a geosynchronous orbit.

For the satellite to be in a geostationary orbit, it must be in an equatorial (inclination = 0°) geosynchronous orbit. If the inclination of the orbit needs to be changed, this is typically done once the satellite is at synchronous altitude, since it requires less propellant, as discussed in Sections 6 and 7.

To place the satellite in the right location in geostationary orbit, proper timing is required. A direct launch to geostationary orbit would need to be timed for the satellite to reach geostationary altitude at the destination position, or the satellite would need to maneuver to its assigned position once in geostationary orbit. By using a parking orbit, the timing for maneuvering into GTO can be separated from considerations that determine the timing of the launch.

AIR LAUNCHING

The mass of the launcher needed to place a satellite in orbit roughly scales with the mass of the satellite, as described above. Consequently, launching small payloads requires a small launch vehicle, and this opens up the possibility of using a rocket that can be carried aloft by an aircraft. A small payload may result from future miniaturization of satellite technology. Or the payload may be a relatively simple system, such as a simple interrogation satellite, a small kill vehicle, or a space mine.

Air-launching has a number of practical advantages. Since the launch does not require a dedicated launch facility, this can in principle reduce costs and allow rapid launches. Since the launcher is mobile, the user can choose the location and latitude of the launch and can reduce restrictions on the direction of launch by, for example, launching over the ocean. This increases the efficiency of getting to orbit and allows a satellite to be launched directly into a desired orbit rather than launching into an orbit determined by the launch site and then maneuvering into the proper orbit.[8]

Since the atmosphere rotates with the Earth, launching eastward from an aircraft allows the launcher to take advantage of the rotational speed of the Earth, just as launching from the ground does.

Since the booster is released above the ground and with an initial velocity equal to that of the aircraft, the requirements on the booster are somewhat reduced. For example, some of the configurations discussed below could increase the booster payload by more than 50% relative to that for the same booster launched from the ground.

8. Small ground-based launchers can have some of these advantages. For example, the SpaceX Falcon 1 launcher is reported to require less launch infrastructure and, therefore, for some missions can be launched from Omelek Island in the Marshall Islands, which lies on the equator. See Craig Covault, "The SpaceX Falcon Will Challenge Orbital Sciences and Boeing," *Aviation Week and Space Technology*, March 28, 2004, http://www.spacequest.com/Articles/The%20SpaceX%20Falcon%20Will%20Challenge%20Orbital%20Sciences%20.doc, accessed January 21, 2005.

Pegasus is an existing air-launched booster that is carried aloft by a B-52 for military payloads or by an L-1011 aircraft for civil payloads. The Pegasus XL has a mass of 23 tons. It can place 450 kg into a 200 km orbit at 28° inclination, 330 kg into a 200-km polar orbit (90° inclination), and 190 kg into an 800-km sun-synchronous orbit. The aircraft releases the three-stage Pegasus booster at an altitude of 12 km at a speed of roughly 0.25 km/s.[9] As of August 2003, Pegasus had been used in 35 launches, the first in 1990.[10]

Other air-launch systems are being developed. The Air Force Research Laboratory is developing a microsatellite launch vehicle (MSLV) that would be launched from an F-15E aircraft, although there are currently no plans to build the system. The goal is a three-stage booster that could place a 100-kg satellite into a 225-km orbit. The aircraft is intended to climb at a 60° angle and release the booster at an altitude of 11.6 km at a speed of about 0.5 km/s. Ultimately, the goal is a 5-ton booster that would be able to place up to 200 kg in a 280-km orbit within 48 hours.[11]

The Defense Department is developing a system called RASCAL (Responsive Access Small Cargo Affordable Launch), which is intended to include an aircraft capable of releasing an expendable booster at much higher altitudes. The aircraft is being designed to release a booster of up to 8 tons at an altitude of 60 km and a speed of 0.37 km/s, with a response time of 24 hours. The booster has not been designed, but the goal is to be able to place roughly 200 kg into a 300-km altitude orbit at low inclinations, or roughly 100 kg into an 800-km sun-synchronous orbit. The first two launches are planned for 2006.[12]

LAUNCH COSTS

The cost of launching satellites into orbit is generally discussed in terms of launch costs per satellite mass, which assumes that the cost roughly scales with the mass of the satellite. While this is not necessarily true, it is a convenient way to estimate launch costs and to compare costs of different launch vehicles. A typical number given for the cost per kilogram of launching objects into low-earth orbit is roughly $20,000 per kilogram ($10,000 per pound).[13] This cost refers to the cost of launching on a large space-launch vehicle such as those shown in Table 8.1.

Not surprisingly, a key goal in developing new launchers is to reduce launch costs. However, there is ongoing debate about what factors drive up

9. Isakowitz et al., 268–279.

10. Orbital Sciences Corporation, "Pegasus Mission History," http://www.orbital.com/SpaceLaunch/Pegasus/pegasus_history.htm, accessed January 21, 2005.

11. William Scott, "Fighters as Spacelift," *Aviation Week and Space Technology*, April 7, 2003, 72.

12. Robert Wall, "Hot Rod to Space," *Aviation Week and Space Technology*, September 22, 2003, 48.

13. See, for example, American Physical Society (APS), Report of the American Physical Society Study Group on Boost-Phase Intercept Systems for National Missile Defense, July 2003, 127, http://www.aps.org/public_affairs/popa/reports/nmd03.html, accessed January 5, 2005.

launch costs and how best to lower them.[14] A second goal is to reduce the time required to launch a satellite. Rapid response is of particular interest to some in the U.S. military, who talk of reducing the time to launch a satellite from weeks or months to hours or days.[15]

One path of development has been reusable launchers, but these will not be available in the near future, and it is unclear to what extent they may reduce launch costs.

A second path is reducing the cost of building and launching conventional launchers. The company SpaceX states that by 2006 its Falcon V launcher, which is similar in size to an Atlas IIA, will be able to place 10 tons in LEO for $20 million ($2,200 per kilogram) and 5 tons into GTO for $20 million ($4,400 per kilogram).[16] Whether such low launch costs are possible remains to be demonstrated.

The miniaturization of satellite technology also permits cost savings. A satellite of a given size can perform more missions, or the same missions can be done with smaller satellites. The latter approach would allow the use of much smaller launch vehicles, which a number of developers believe would reduce launch costs.[17] For example, the SpaceX Falcon I booster under development has a launch mass of only 30 tons, which is much smaller than the vehicles listed in Table 8.1. The goal is to launch satellites of 600 to 700 kg to LEO for $6 million (corresponding to about $10,000 per kilogram). It is also intended to provide rapid response and be ready to launch in 24 hours.[18] Similarly, the Microcosm Sprite vehicle is designed to place a 300 kg satellite in LEO for $1.8 million ($6,000 per kg).[19] A current goal for the RASCAL launcher is to place a 75-kg payload in orbit for $750,000 ($10,000 per kg).[20]

The Orbital Sciences Minotaur and air-launched Pegasus vehicles currently launch small payloads, but at a cost of $40,000 to $50,000 per kilogram—significantly higher than the costs projected for the Falcon I and Microcosm Sprite.[21]

In addition, sufficiently small satellites can be small enough to piggyback on another satellite's launch, often leading to substantially reduced launch costs.

14. See, for example, Peter Taylor, "Why Are Launch Costs So High?" September 2004, http://www.ghg.net/redflame/launch.htm, accessed January 21, 2005.

15. William Scott, "Rapid Response," *Aviation Week and Space Technology*, April 7, 2003, 66.

16. Covault.

17. See, for example, Matt Bille and Robyn Kane, "Practical Microsat Launch Systems: Economics and Technology," Paper SSCO3-III-3, AIAA/USU Conference on Small Satellites, August 2003, http://www.mitre.org/work/tech_papers/tech_papers_03/kane_mls/kane_mls.pdf, accessed January 21, 2005.

18. SpaceX, http://www.spacex.com, accessed January 21, 2005; Covaut; Michael Dornheim, "Quick, Cheap Launch," *Aviation Week and Space Technology*, April 7, 2003, 70.

19. Dornheim.

20. Leonard David, "Military Space: Securing the High Ground," Space.com, April 2, 2003, http://www.space.com/businesstechnology/technology/higher_ground_030402.html, accessed January 21, 2005.

21. Minotaur is reported to place over 400 kg in LEO at a cost of up to $19 million (Bille and Kane). Pegasus is reported to cost $22–26 million to place 500 kg in LEO (Dornheim).

Section 8 Appendix A: Potential and Kinetic Energy of Satellites

The potential energy of a satellite is a measure of the energy required to lift it to its orbital altitude, whereas the kinetic energy reflects the amount of energy required to give the satellite its orbital speed.

For a circular orbit at altitude h, the kinetic energy of a mass m due to its orbital speed is

$$KE = \frac{1}{2} m V^2 = \frac{GM_e m}{2r} \tag{8.1}$$

where $r = h + R_e$ and the second equality uses Equation 4.2 for the orbital speed.

It is useful to discuss the potential energy in two ways. The first sets the zero of potential energy at the Earth's surface, since this is useful in comparing how much kinetic versus potential energy a satellite gains by being placed into orbit.

With this choice, the potential energy of a mass m at an altitude h is

$$PE = GM_e m \left(\frac{1}{R_e} - \frac{1}{R_e + h} \right) = GM_e m \frac{h}{R_e r} \tag{8.2}$$

The ratio of potential to kinetic energy of a mass in a circular orbit is then $h/2R_e$. Thus, kinetic energy dominates potential energy out to $h = 2R_e =$ 12,740 km.

The total energy of a mass in circular orbit at altitude h is

$$PE + KE = \frac{GM_e m}{2R_e} \left(\frac{2\frac{h}{R_e} + 1}{\frac{h}{R_e} + 1} \right) = \frac{1}{2} m V_{escape}^2 \left(\frac{1 + \frac{h}{r}}{2} \right) \tag{8.3}$$

where V_{escape} is the escape velocity. The last expression shows that the energy is bounded by the kinetic energy the object would have if its speed was equal to the escape velocity.

If instead the zero of potential energy is set at infinite distance from the Earth (i.e., outside the gravitational well of the Earth), the potential energy is given by

$$PE = -\frac{GM_e m}{r} = -2KE \tag{8.4}$$

for all r, which is a special case of the Virial Theorem.

Section 8 Appendix B: The "1/2 Rule"

The "1/2 Rule" states that a ballistic missile that can carry a given payload to a maximum range R on the Earth can lift that same mass to an altitude of roughly $R/2$ when launched vertically.

For the case of a missile launched on a flat earth, it is straightforward to show that the 1/2 Rule is exact. As a result, it holds for short-range missiles, for which the curvature of the Earth can be neglected.

In the flat-earth approximation, the gravitational acceleration is constant with altitude. Consider a missile of mass m fired vertically with a speed V. The maximum height h it reaches is found by equating kinetic and potential energy:

$$\frac{1}{2}mV^2 = mgh \Rightarrow h = \frac{V^2}{2g} \tag{8.5}$$

The time it takes to reach its apogee is

$$h = \frac{1}{2}gt^2_{apogee} \Rightarrow t_{apogee} = \frac{V}{g} \tag{8.6}$$

To maximize missile range on a flat earth, the missile is launched at 45°. If the initial missile velocity is V, the vertical and horizontal components (V_v and V_h) are both equal to $(V/\sqrt{2})$. The missile's range is then given by the (constant) horizontal speed multiplied by the time it takes the missile to climb to apogee and fall back to Earth:

$$range = 2t_{apogee}V_h = 2\left(\frac{V_v}{g}\right)V_h = \frac{V^2}{g} \tag{8.7}$$

which is twice the maximum height found above.

For the round-earth case, the gravitational acceleration varies with altitude. The maximum altitude a missile can reach when fired vertically is estimated by again setting the potential energy at the maximum altitude h equal to its initial kinetic energy. Assuming the missile has a speed V at the Earth's surface, then

$$GM_e m\left(\frac{1}{R_e} - \frac{1}{R_e + h}\right) = \frac{1}{2}mV^2 \tag{8.8}$$

Solving for the maximum altitude h gives

$$h = R_e\left(\frac{(V/V_0)^2}{2 - (V/V_0)^2}\right) \tag{8.9}$$

where

$$V_0 \equiv \sqrt{\frac{GM_e}{R_e}} = 7.91 \text{ km/s} \tag{8.10}$$

is the orbital speed of a circular orbit with a radius equal to R_e.

This equation shows that $V = \sqrt{2}\,V_0 = 11.2$ km/s gives $h = \infty$ and is therefore the escape velocity from the Earth. It also shows that $V = V_0$ gives $h = R_e = 6370$ km.

For V = 3 km/s, which corresponds to a 1,000-km range ballistic missile, this equation gives $h = 0.078R_e$ = 495 km, and for V = 7.2 km/s, which corresponds to a 10,000-km range ballistic missile, this equation gives $h = 0.71R_e$ = 4525 km. These results show that the 1/2 Rule holds for missiles with range small compared to the radius of the Earth and continues to hold approximately even for longer ranges.