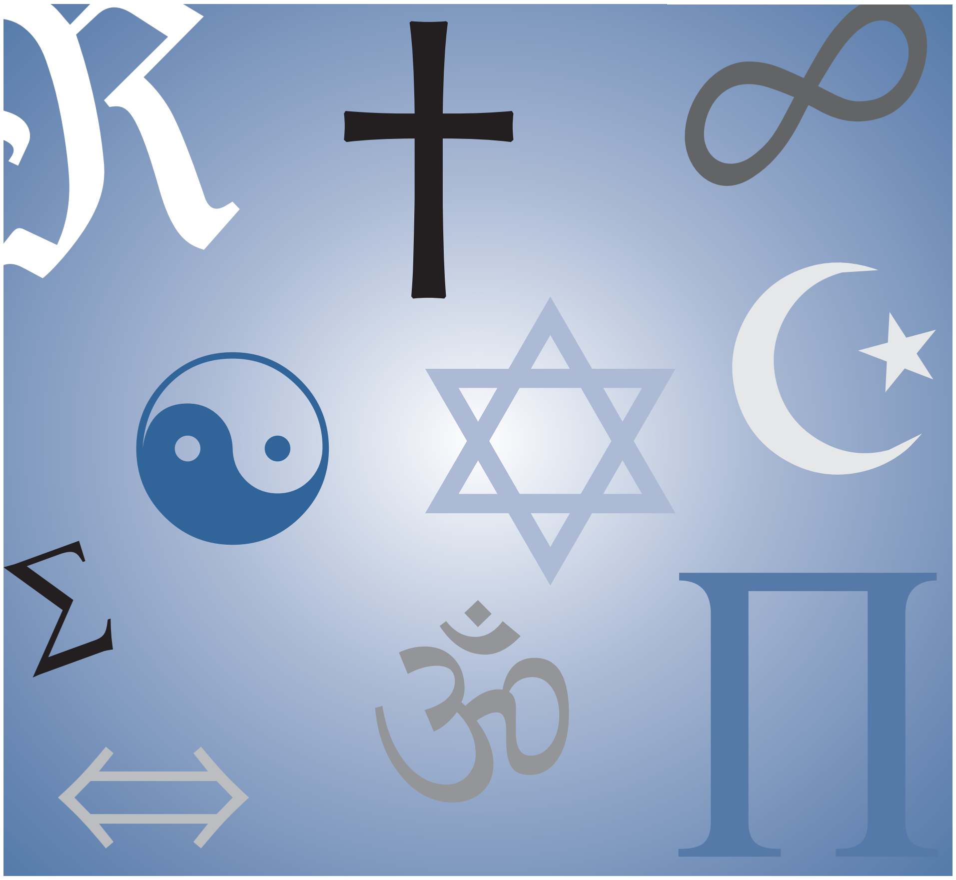

Russian Religious Mystics and French Rationalists: Mathematics, 1900–1930

Loren Graham and Jean-Michel Kantor

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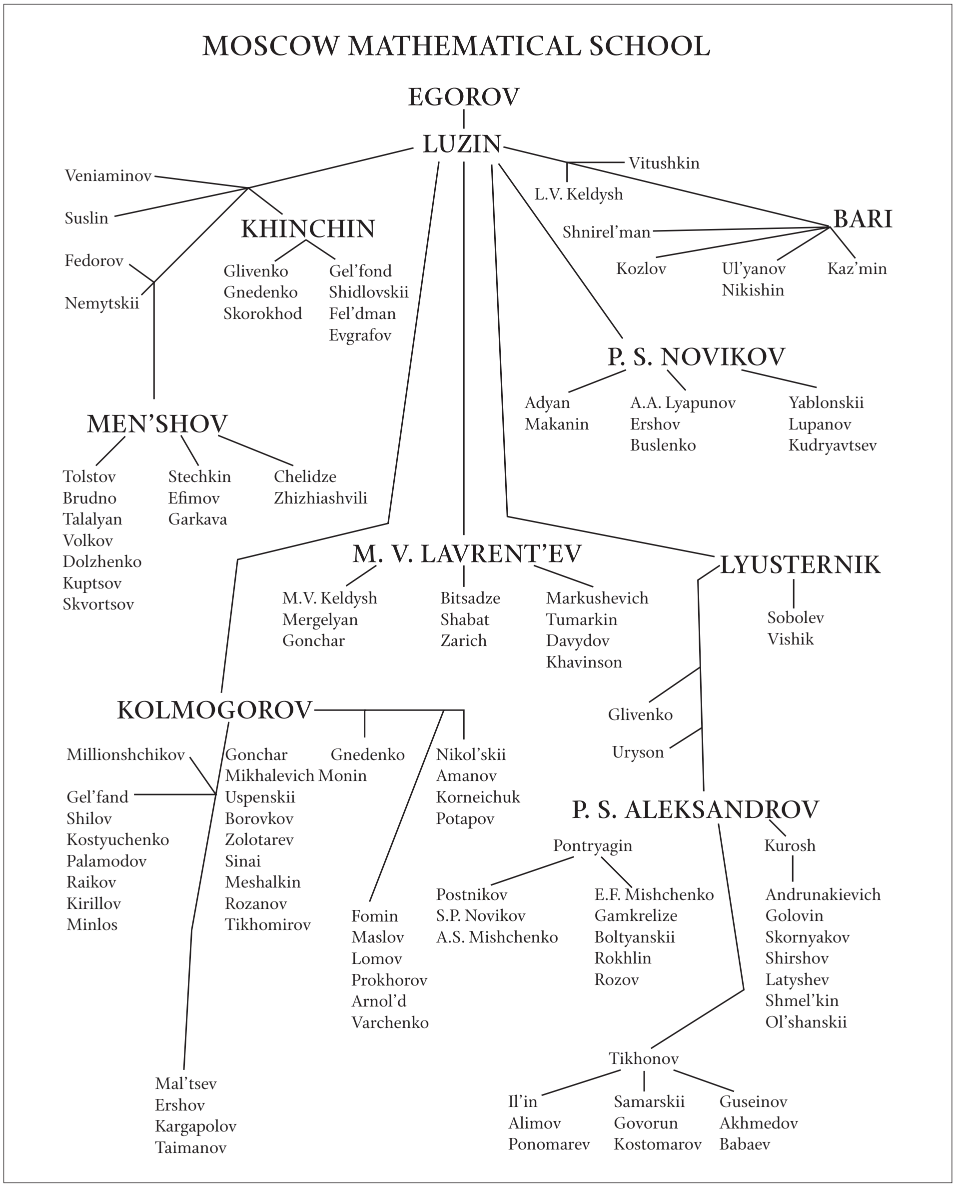
Loren Graham

If someone were to ask me what I think was the greatest intellectual contribution that Russians made in the twentieth century, I would answer, without much hesitation, mathematics and fields closely connected with it, such as theoretical physics. The Moscow School of Mathematics was one of the most influential movements in twentieth-century mathematics. In particular, the study of functions and the descriptive theory of sets (the application to real numbers of set theory), initiated by Dmitrii Egorov, Nikolai Luzin, and their students in the first decades of the twentieth century, has had a worldwide impact.

If you go today to the mathematics department of Moscow University, where this movement began, you might see on a bulletin board, as I have seen, a genealogical chart depicting the founders of this impressive mathematical movement and their succeeding generations of students. Mathematicians or those familiar with the world of mathematics would recognize the names of some of the most influential mathematicians of the last century: for example, Andrei Kolmogorov, perhaps the greatest probabilist of the twentieth century; Sergei Novikov; Vladimir Arnol'd; Lev Pontriagin; Pavel Aleksandrov; and Mstislav Keldysh, onetime president of the Soviet Academy of Sciences and the theoretician of the Soviet space program.

At the top of the genealogical tree are Egorov and Luzin. Who were these men? Where did they come from? What motivated them? How did they differ from other leading mathematicians, especially the French who were at the time considered pioneers in the same fields? My colleague from Paris, Jean-Michel Kantor, and I have been investigating these questions, and we have come to a conclusion that surprises us and runs counter to our own secular predispositions: at the heart of the birth of the Moscow School of Mathematics was a mystical religious impulse. This mystical doctrine was defined by the established Russian Orthodox Church as a heresy and hence condemned. Yet the heresy, known as Name Worshipping (*imiaslavie*), never died and it even has a small life in Russia today; indeed, it has gained some strength in recent years. Several outstanding mathematicians are involved with it at present, but their interest in it remains hidden, as it always has been.

During the last two years, I have been in Moscow a number of times, and I have visited with Russian scholars who are familiar with the Name Worshipping movement. One of them was a mathematician – a rather well known one whom I prefer not to name in order to preserve his privacy. I knew that he was philosophically and religiously interested in Name Worshipping, and so I asked if it would be possible to witness Name Worshippers practicing their faith. His answer was no: "Name Worshipping is an intimate practice that is best done alone." I asked if there was any place where Name Worshippers particularly liked to worship. He replied that Name Worshipping cannot be done openly in established churches or cathedrals because the official church disapproves of the practice. He added, however, that there was one place particularly sacred to Name Worshippers: the basement of the Church of Saint Tatiana the Martyr in Moscow. I knew where this church was. Before the Russian Revolution it was the official church of Moscow University, and now that the Soviet Union has disappeared, it has become so again. For many years the mathematics department of the university was located next to it. During the Soviet years it was converted into a sort of student club, and one time in the early 1960s I went to a dance there with my young wife,



Patricia. We did not know at the time that we were dancing in what had once been a church, nor did we have any idea that this place would become important to my research.

I asked the Moscow mathematician how I would know when I had reached the spot sacred to Name Worshippers. He told me that I would know when I got there. I went there this past July and wandered around, searching the whitewashed walls of the basement. Then I found a peculiar corner, and I knew immediately I was in the right place. On the walls were two photographs of the two men who were instrumental in establishing Name Worshipping among mathematicians, namely, Dmitrii Egorov and Pavel Florenskii.

Dmitrii Egorov (1869 – 1931) and Nikolai Luzin (1883 – 1950) founded the Moscow School of Mathematics. They had close connections with French and German mathematicians. Egorov spent the year of 1902 in Paris, Berlin, and Göttingen and talked with, among others, the mathematicians Henri Lebesgue, Henri Poincaré, Jacques Hadamard, and Kurt Hensel. Luzin first went to western Europe in 1905, later visited France and Germany a number of times, and had frequent contacts with mathematicians there.

In the first years of the twentieth century, Luzin studied mathematics in Moscow University under Egorov and as a fellow student with Pavel Florenskii (1882 – 1937), who were influential in forming the ideas of the Moscow School. In their mature and professionally active years, all three men – Florenskii, Egorov, and Luzin – were deeply religious. Florenskii, disappointing his teachers, abandoned mathematics for religious studies and became a priest. Egorov and Luzin went on to become outstanding mathematicians who helped create an explosion of mathematical research in Moscow in the 1920s and early 1930s. Florenskii and Egorov would eventually be arrested by the Communist authorities, accused of mixing mathematics and religion. They subsequently died in prison. (Parenthetically I would observe that it is one of the cruel ironies of history that the Communists' charge that Florenskii and Egorov mixed mathematics and religion was correct; although contrary to the assumption of the Communists, the mixture was amazingly fruitful to the field of mathematics.) Luzin narrowly escaped imprisonment, even though he was put on "trial" for ideological deviations and severely reprimanded. Florenskii is credited with developing a new ideology of mathematics and religion that played a role in the pioneering mathematics work of Egorov, Luzin, and their students.

Florenskii was one year older than Luzin and entered Moscow University in 1900; Luzin followed him in 1901. Both studied with Egorov, who was a young professor of mathematics. Luzin at that time was not the religious believer that he later became. By his own admission he was a "materialist," like many other young Russian intellectuals, and he knew very little about philosophy or politics.

From 1905 to 1908 Luzin underwent a psychological crisis so severe that several times he contemplated suicide. One precipitating event in Russia was the unsuccessful revolution of 1905, a moment that sobered many left-wing members of the intelligentsia who had talked romantically of their hopes for a revolution without comprehending the blood and violence that revolutions often bring. Shocked by the suffering, a number of intellectuals, in both the natural and social sciences, began to rethink their positions.

Luzin possessed a tender, somewhat naive personality, and he was not prepared for the pain he saw around him during and immediately after the revolutionary events. In an effort to relieve his spiritual crisis, his teacher Egorov sent him abroad in December 1905, but the trip did not solve Luzin's spiritual and intellectual problems. Not only did Luzin's materialist worldview collapse, but his faith in science and mathematics did as well. He was totally without a purpose in life. In despair on May 1, 1906, he wrote Florenskii from Paris:

> You found me a mere child at the University, knowing nothing. I don't know how it happened, but I cannot be satisfied any more with the analytic functions and Taylor series. . . . To see the misery of people, to see the torment of life . . . this is an unbearable sight. . . . I cannot live by science

alone. . . . I have nothing, no worldview, and no education. I am absolutely ignorant of the philological sciences, history, philosophy.

In a long correspondence and in numerous meetings at Sergeev Posad, Zagorsk, a monastery town outside Moscow, Florenskii, already a devout believer, supplied Luzin with a new worldview. It combined both religion and mathematics and, as we will see, gave the desperate Luzin reason to believe that he could renew his mathematical research while at the same time serving moral and religious purposes.

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Many of the ideas Luzin found stimulating and reassuring were presented in an essay Florenskii wrote in 1903, when he was only twenty-one years old and still a mathematics student at Moscow University. In this essay, entitled "The Idea of Discontinuity as an Element of World View," Florenskii displayed a characteristic that was very common among members of the Russian intelligentsia of his time: the belief that all intellectual life forms a connected whole and that therefore ideas in mathematics and philosophy could be extended to the social and moral realms, and vice versa.

Florenskii thought that much of the nineteenth century had been a disaster from the standpoint of philosophy, religion, and ethics and that the particular type of mathematics that reigned during that century was one of the important causes of this misfortune. The governing mathematical principle of the nineteenth century, which Florenskii saw as responsible for ethical decline, was deterministic "continuity": the belief that all phenomena pass from one state to another smoothly. In substitution of this "false" principle of continuity, Florenskii proposed its opposite, discontinuity, which he saw as morally and religiously superior. The nineteenth century was, according to him, the unfortunate apogee in faith in deterministic continuity; indeed, he wrote that in the nineteenth century "the cementing idea of continuity brought everything together in one gigantic monolith." The mathematical approach that created this monolith was infinitesimal analysis and differential calculus. This method became all-powerful because differential calculus was at the heart of the physical sciences through Newtonian mechanics. One of the results of its seeming omnipotence was that mathematicians concentrated only on continuous functions, since "continuous functions are differentiable" and therefore susceptible to analysis by the calculus.

Florenskii believed that, as a result, mathematicians and philosophers tended to ignore those problems that could not be analyzed by calculus, namely, the discontinuous phenomena. Seeing continuous functions in mathematics as "deterministic," Florenskii believed the expansion of the philosophy of determinism throughout psychology, sociology, and religion was the destructive result of a temporary emphasis in mathematics. Thus he held nineteenth-century mathematics responsible for the erosion of earlier beliefs in freedom of will, religious autonomy, and redemption.

Florenskii thought that the field that was "guilty" of the glaring overestimation of continuity – mathematics – was destined to lead thinkers out of the blind alley that it had created. In the 1880s the German mathematician Georg Cantor, the founder of set theory, had analyzed "continuum" as merely a set among possible other sets and had therefore deprived the concept of its metaphysical, dogmatic power. Now the road was open, maintained Florenskii, to restore discontinuity and indeterminism to their rightful place in one's worldview. He saw the power of discontinuity in recent developments in many fields outside mathematics, such as the theory of mutations in biology (delivering, according to Florenskii, biology from the "heartless" continuity of Darwinism), new ideas about molecular physics, and concepts of "subliminal consciousness" and "creativity" in psychology. Surveying these developments, Florenskii called for "the dawn of a new discontinuous worldview" and challenged his mathematician colleagues, such as Luzin and Egorov, to foster this new approach, one that would combine mathematics, religion, and philosophy.

In the years just before the Russian Revolution of 1917, the world of Russian Orthodoxy, the state religion, was shaken by a theological struggle that further influenced Florenskii, Luzin, and Egorov and their ideas about the relationship between mathematics and religion. A polemic developed between two groups of religious believers: the Worshippers of the Name, or Nominalists (*Imiaslavtsy*), and the Anti-Nominalists (*Imiabortsy*). The dispute was rooted in an ancient question about how humans can worship an unknowable deity. If God is in principle beyond the comprehension of mortals (and holy scripture contains many such assertions), how, in complete ignorance of his nature, can human beings worship him? What does one worship? The most common response given to this dilemma throughout religious history was the resort to symbols: icons, names, rituals, music, relics, scents, tastes, art, architecture, literature. Symbolism is the term given to a perceptible object or activity that represents to the mind the semblance of something that is not shown but realized by association with it.

Mathematical objects cannot be shown so both religion and mathematics make heavy – but different – use of symbols. Mathematics uses such symbols as:

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Religion uses a great variety of symbols, such as the Star of David and the cross, as well as icons, prayers, chants, and hymns.

But some questions naturally arise: What reality, if any, lies behind the symbol? What does a religious icon or a mathematical symbol really represent? Does the symbol acquire any sort of autonomy?

The issue of religious symbols took on an unusual sharpness in Russia in the years 1908 – 1930, the same years in which the Moscow School of Mathematics was created. Priests and mathematicians were involved in both the religious and the mathematical discussions. In 1907 a monk of the Orthodox Church, Ilarion, who had earlier spent years in a Russian monastery in Mount Athos in Greece, published a book, *In the Mountains of the Caucasus*, that seized on an existing

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tradition in Orthodox liturgy, especially the chanting of the Jesus Prayer (*Iisusova molitva*), and raised it to a new prominence. In the Jesus Prayer, the religious believer chants the names of Christ and God over and over again, hundreds of times, until his whole body reaches a state of religious ecstasy in which even the beating of his heart, in addition to his breathing cycle, is supposedly in tune with the chanted words "Christ" and "God." (A state vividly described by J. D. Salinger in *Franny and Zooey*.) According to Ilarion, the worshipper achieves a state of unity with God through the rhythmic pronouncing of his name. This demonstrates, said Ilarion, that the name of God is holy in itself, that the name of God is God (*Imia Bozhie est' sam Bog*).

At first this book was well received by many Russians interested in religious thought. Ilarion's views became very popular among the hundreds of Russian monks in Mount Athos, who gradually spread the views elsewhere. But the highest officials in Russian Orthodoxy, in Saint Petersburg and Moscow, soon began to consider the book not just as a description of the reality of prayer but as a theological assertion. For many of them, the adherents of Ilarion's beliefs were heretics, even pagan pantheists, because they allegedly confused the symbols of God with God himself. On May 18, 1913, the Holy Synod in Saint Petersburg condemned the Name Worshippers; soon thereafter the Russian Navy, with the approval of Tsar Nikolai II, sent several ships (the Donets and the Kherson) to Mount Athos to bring the rebellious monks forcibly to heel. Over six hundred unrepentant monks were flushed out of the monastic cells with fire hoses, arrested, and brought under guard to Odessa. In later detentions, the number grew to approximately one thousand. The dissidents strongly protested their treatment and obtained promises of further investigation and reconsideration.

With the advent of World War I, the issue receded into the background, but until the end of the tsarist regime, the adherents of the "heresy" were forbidden to return to Mount Athos or to reside in major cities like Saint Petersburg and Moscow. The most fervent of them retreated to monasteries, where they continued to practice their variant of the faith. After the Bolshevik Revolution in October and November of 1917, the Name Worshippers, now living all over rural Russia, were more successful than most other religious believers in continuing their practices out of view of Soviet political authorities, who were trying to suppress religion. After all, the Name Worshippers had already been defined as heretics and excluded from the established churches. But in secret they continued their faith, and as a result they were not compromised by association with the Bolsheviks, as some of the established church leaders soon became. The dissidents claimed to be representatives of the undefiled "true faith," increasing their popularity with some religious opponents of the new Communist regime.

In the 1920s the German writer and journalist Rene Fulop-Muller spent much time in Russia and in his remarkable book *The Mind and Face of Bolshevism*, he wrote that Name Worshipping was "a movement to which a great part of the intelligentsia as well as a considerable part of the peasantry belong. The best men of Russia lead this school, which proclaims the magic power of the divine name."

After the Bolshevik Revolution, Florenskii lived in Sergeev Posad, Zagorsk, and he was close religiously and intellectually to the Name Worshipper dissidents. He communicated their ideas to Luzin and Egorov, his mathematician colleagues, and he translated these religious concepts into mathematical parlance. In the early 1920s, there was a Name Worshipper Circle (*imeslavcheskii kruzhok*) in Moscow where the ideas of the religious dissidents and the concepts of mathematics were brought together. Florenskii and the philosopher A. F. Losev attended meetings of the circle, which included fifteen or sixteen philosophers, mathematicians, and religious thinkers. Sometimes the circle met at Egorov's apartment and Florenskii gave papers at the meetings. At these meetings, Florenskii maintained that "the point where divine and human energy meet is 'the symbol,' which is greater than itself." To Florenskii, religious and mathematical symbols could attain full autonomy.

Florenskii saw that the Name Worshippers had raised the issue of "naming" to a new prominence. To name something was to give birth to a new entity. God said in Genesis, "Let there be Light, and there was Light." He named it first, and then He created it. Names are words. In the Gospel according to Saint John, the statement occurs, "In the beginning was the Word, and the Word was with God, and the Word was God." Florenskii believed that mathematicians who created new entities like sets by naming them came as close as humans are permitted to approaching the divine.

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The famous sentence of Georg Cantor, "The essence of mathematics lies precisely in its freedom," clearly had a strong appeal to Florenskii. In mathematics, more than in the threatening Soviet world he was facing, men like Florenskii could exercise their free will and create beings (sets) by just naming them. For example, defining the set of numbers such that their squares are less than 2, and naming it "A," and analogously the set of numbers such that their squares are larger than 2, and naming it "B," immediately brought into existence the real number $\sqrt{2}$ (essentially the Cauchy construction).

The development of set theory was to Florenskii a brilliant example of how renaming and reclassifying can lead to mathematical breakthroughs. A "set" was simply a renaming of

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entities according to an arbitrary mental system, not a recognition of the types of real material objects. When a mathematician created a set by naming it, he was giving birth to a new mathematical being. The naming of sets *was* a mathematical act, just as the naming of God *was* a religious one, according to the Name Worshippers. A new form of mathematics was coming, said Florenskii, and it would rescue mankind from the materialistic, deterministic modes of analysis so common in the nineteenth century. Indeed, set theory and new insights on continuous and discontinuous phenomena became hallmarks of the Moscow School of Mathematics.

Leading mathematicians everywhere at this time were wrestling with the problem of what is allowed in mathematics and what is to be considered a good definition of a mathematical object. As the French mathematician Lebesgue wrote to his colleague Emile Borel in 1905, "Is it possible to prove the existence of a mathematical object without defining it?" To Florenskii the question was the analogue of, "Is it possible to prove the existence of God without defining him?" The answer for Florenskii and later for Egorov and Luzin was that the act of naming in itself gave the object existence. Thus naming became the key to both religion and mathematics. The Name Worshippers gave existence to God by naming him and worshipping him, and mathematicians gave existence to sets by naming them and working with them. The Russian mathematicians asked, for example, "How can we know that there are numbers greater than infinity – transfinite numbers – if infinity is defined as the largest possible number? We know because we can name them – we call them 'aleph numbers' – and we work with them."

The idea that naming is an act of creation goes back very far in religious and mythological thought. The claim has been made that the Egyptian god Ptah created with his tongue that which he conceived. In the Jewish mystical tradition of the Kabbala (Book of Creation, Zohar), there is a belief in creation through emanation, and the name of God is considered holy.

The connection between the religious dissidents in Russia and the new trends in Moscow mathematics went beyond the suggestions and implications so far discussed. There was a direct linguistic connection. The Moscow mathematicians Luzin and Egorov were in close communication with French mathematicians with similar concerns. Lebesgue introduced in 1905 the concept of "effective sets," and he spoke of "naming a set" (*nommer un ensemble*); such a set was then often called a "named set" (*ensemble nommé*). The Russian equivalent was *imennoe mnozhestvo*. Thus the root word *imia* (name) occurred in the Russian language in both the mathematical terms for the new types of sets and the religious trend of *imiaslavie* (Name Worshipping). Indeed, much of Luzin's work on set theory involved the study of effective sets (named sets). To Florenskii this meant that both religion and mathematics were moving in the same direction.

Jean-Michel Kantor

The French mathematicians were not ready for the new mathematics that occurred with the birth of set theory. They were rather skeptical of this "German metaphysics" founded by Georg Cantor. If we want to give an overview of French reaction at this time, in order to compare their attitude with that of the Russians, we need to comprehend a very different cultural context. The French cultural milieu is strongly marked (through centralized education, for example) by at least three different influences.

First, there is the old cultural tradition of Cartesianism: *Le primat de la raison*. *Penser* (to think), this is the main activity in science. The main activity of thought is *la raison* (reason). One can think about mathematics (*penser les mathématiques*); it is not purely formal logic, as Bertrand Russell would say later. If I can think about a mathematical notion, then it exists; conversely, if I cannot think of it, it surely does not exist.

This is a very important concept for Lebesgue and Borel, who could not think of nondenumerable infinities and so denied their existence (after a short period of juvenile enthusiasm by Borel). It accounts for their reservation about the Russian approach (see, for example, Lebesgue's description of Luzin's "philosophical" mind in the preface of his 1930s book). Also important is the tradition of the Cartesian method as described by René Descartes: If you have a problem, just cut it into parts as long as you can and you'll solve the problem.

A second strong influence is Auguste Comte's positivism. Science cannot reach the primal causes (*les causes premières*) but can, after liberating itself from all metaphysical influence and any theological tendency, reach a perfect form of discourse. Comte's philosophy builds a wall between the metaphysical and the scientific order of things. Once science enters the "positive stage," its goal is no longer a metaphysical quest for truth nor a rational theory purporting to represent reality. Science is composed of laws, not theories. Laws are correlations of observable facts that we need in order to predict. Mathematics, through the theory of functions (an old French tradition going back to Joseph-Louis Lagrange and Charles Fourier), is suitable for the analysis of natural phenomena via the laws of physics expressed since Isaac Newton by differential equations.

A third, more subtle, factor is Blaise Pascal's *esprit de géométrie*. I remind you that geometry for Pascal is much broader than what we imagine today as geometry. The universal, unique, human truth comes from geometry through *la lumière naturelle*, a very religious approach in Pascal, but also very deeply involved in philosophy, without being ever able to reach the deepest of things. Geometry's real content allows one to distinguish between nominal and real definitions (*la définition de noms et la définition de choses*).

Georg Cantor (1845 – 1918) created set theory around 1870. It started with a revolutionary definition of infinities, the first new step since Aristotle (384 – 322 B.C.) distinguished between potential and actual infinities in his *Physics*, denying that the actual infinite exists and allowing only the potential infinite. Cantor gave it a name; he called it the first infinite "aleph-zero," the denumerable, the number so to say of all integral numbers: 0, 1, 2 . . . . Galileo had already noticed that there are just as many integers as there are even integers: that is, there are just as many 1, 2, 3, 4, 5, as there are 2, 4, 6, 8. Cantor turned this apparent contradiction into a definition of what is an "infinite" set. He defined many more infinite numbers. It is interesting to notice at this point that the creation of these alephs was very close in Cantor's mind to the creation of irrational numbers starting from rational numbers, which allows a precise mathematical definition of what we call today the continuum – the continuum of space or of lines. Applying his new theory to the continuum – the real line, the set of all real numbers – was the next revolutionary step. Was this allowed? How the continuum could be made out of points, like matter from atoms, was an issue at the time.

German mathematician Paul Du Bois-Reymond (1831 – 1889) had already rejected a part of the new set theory. He accepted "actual infinite" but rejected the philosophy of the continuum (points on the line or points of our space) as presented by Cantor. For Cantor the continuum was a reduction of continuous quantities to discrete entities; for Du

For Cantor the continuum was a reduction of continuous quantities to discrete entities; for Du Bois-Reymond the continuum had a mystical nature outside of mathematical knowledge.

Bois-Reymond the continuum had a mystical nature outside of mathematical knowledge. This direction of thought would be developed further by Herman Weyl (1885 – 1955) and Jan Brouwer (1881 – 1966), leading to an important current in mathematical thought called intuitionism.

A natural question to ask was, is there another infinite between aleph-zero and the power of the continuum? This is the famous Continuum Hypothesis, stated as the first problem in the famous list of problems given by David Hilbert (1862 – 1943) at the Paris International Congress of Mathematicians in 1900 under the title "*Problème de M. Cantor relatif à la puissance du continu*."

Since 1878, the main purpose of Cantor's research had been to prove the Continuum Hypothesis, which led (through important results in analysis) to the birth of descriptive set theory. His strategy was to invent and construct more and more complicated subsets of the continuum. For example, he invented the "Cantor ternary set," which he defined in an endnote to Grundlagen in 1883: It is the limit of the sets obtained by taking one out from one-third intervals at each step. It is equal to its set of accumulation points, not isolated points; it does not contain any interval; and it has "the power of the continuum" (number of elements).

On September 26, 1904, Ernst Zermelo (1871 – 1953) wrote to Hilbert, telling him that he had developed a proof that in any set there is a way to put all elements in a good order (*Beweis, dass jede Menge wohlgeordnet worden kann*): that is, an order with essentially the same properties as the order of positive integers. In the proof, he used a fact that would later be called the Axiom of Choice: for any family of nonempty sets, there exists a way to associate one particular element to each of these sets. Of course, one may ask what is meant by "associate" and "particular element." After Zermelo's declaration, the fight began!

The debate was especially strong in France, where most of the important young mathematicians exchanged strong-worded letters. The five letters that Baire, Borel, Lebesgue, and Hadamard exchanged in 1905 describe the point of view of the most active young mathematics leaders with respect to the new set theory.

The men who faced the new mathematics were very different in character as well as in social personalities. Henri Poincaré (1854 – 1912) was the master of French mathematics, the last universal mathematician, and a philosopher of mathematics. René Baire (1874 – 1932) came from a very poor family in the region of Beauvais. He had a strict, serious life. He taught in colleges for most of his career and suffered from psychosomatic diseases, with his life ending very sadly. Emile Borel's (1871 – 1956) life is a typical success story of the French intellectual elite of the Third Republic. He was a brilliant, successful mathematician, a journalist, and an active participant in the Parisian scene. At the same time, Borel had strong country roots: his father was a protestant priest in the southwest (Rouergue).

For Borel, numbers had a reality almost like flesh. He required that mathematics provide Cartesian evidence that was as close to the sensual as to the rational. This is why he later abandoned mathematics when he realized that set theory was taking a path too abstract for him.

Henri Lebesgue (1875 – 1941) was a passionate, pure spirit; more precisely, he was an aristocrat of geometry. Lebesgue and Borel had a long friendship based on mutual ad-

The issues mixed philosophy, linguistics, psychology, and mathematics and the results were too much to handle. How would one separate and use the Cartesian method?

miration. But Lebesgue looked for quarrels concerning intellectual priorities, and their friendship ended with a remarkable sad letter of farewell from Lebesgue: "I kept too much hidden friendship for you not to be sad about my current state of mind." Both Baire and Lebesgue have left their names in the domain called analysis; both had a strong obsession with rigor inherited from the school of Cauchy.

Lebesgue, Baire, and Borel did not anticipate the events of 1900 and 1904 in Paris and then in Germany. The French mathematician Jacques Hadamard (1865 – 1963) accepted the new axiom, while Lebesgue, Baire, and Borel essentially opposed the consequences of the axiom. Borel later published articles and books about set theory and applications, trying to explain fifty years of varying opinions concerning set theory.

The axiom discussion centered on what could be done in mathematics, how mathematical beings could be defined in order to be accepted in the process of mathematics, and what was a good definition. Among the motivations for this attitude, I mention the Cartesian principle of separating the problems, the disciplines, and the absolute truth of mathematics. As Borel put it: "We are serious people; this at least is not philosophy; a disagreement can only be due to a misunderstanding." But what is allowed in mathematics? Here are a few sentences from Lebesgue about the Axiom of Choice: "If you have to choose in a set, you talk about objects as if they were in a bag, and you know nothing about them. You just know they have a certain property, which other elements in the bag don't have. So you cannot define any order about the elements."

This is the French approach to ontological issues. As Lebesgue put it, "What we say has only some meaning if precise laws are given, if we apply our reasonings to precise data."

We come close to metamathematics, and you meet the two opposite schools. These schools fight together, like the scholastics in the Middle Ages, and discuss what meaning to give to the word 'existence' in mathematics.

The discussion about the Axiom of Choice was lively for yet another reason. If the Axiom was accepted, many consequences would follow from it, even in the familiar realm of geometry (such as the Hausdorff paradox, which led, in 1924, to a surprising fact in geometry called the Banach-Tarski paradox).

As a result of this discussion, French mathematicians limited themselves, for example, to the first infinite (aleph-zero). Borel, in a typical Cartesian attitude, would not accept big infinities if he could not imagine them or think of them. Lebesgue called a set "*nommé*," and then later "*ensemble effectif*," where there would be no construction using the existence of a Zermelo correspondence.

The issues mixed philosophy, linguistics, psychology, and mathematics and the results were too much to handle. How would one separate and use the Cartesian method? Take, for example, Richard's paradox, which appeared in 1905. Richard was a young, provincial math teacher in Dijon who wrote an article in which he described, in a simple way, a number given by a seemingly paradoxical definition: For example, call N the smallest number that could not be described with less than thirty words in English. Now I just defined a number that has been defined by the sentence above. The definition defines it, although it cannot be defined!

This mixing of fields was frightening to the French. For example, in 1919, reporting on Lebesgue's work, Paul Appell (Borel's father-in-law and a very powerful mathematician) wrote, "We come close to metamathematics, and you meet the two opposite schools. These schools fight together, like the scholastics in the Middle Ages, and discuss what meaning to give to the word 'existence' in mathematics." Now the same word can be found with a big "E" and with very different tonality in Luzin's manuscripts, but not with the same connotations.

Incidentally, Luzin's manuscripts, copied by R. Cooke in 1979 and not yet completely analyzed, reveal dramatic efforts including psychological approaches to mathematical issues. I quote: "Everything seems to be a day-dream, playing with symbols, which, however, yield great things." French mathematicians limited the direct search into the *gouffre du continu*, the black hole of the continuum. Other constructions, more down-to-earth, with numbers defined by decimal expansions, were proposed by Borel. For example, he was interested in a concrete definition of normal numbers in connection with probability and measure theory. But the French mathematicians still used set theory for the classification of functions, as in a remarkable text of Lebesgue's in 1905, where he defined a new class of function called "analytically representable."

The new field of mathematics that resulted from first the trials of the French school and then Luzin's work, which would be called descriptive set theory, can be assigned a precise birth day: the day Mikhail Suslin (1894–1919), a young student, rushed to see his thesis-advisor, Luzin, to show him the mistake he had found in a ten-year-old seminal article of Lebesgue's. This famous mistake has been the subject of much discussion. It is neither subtle nor trivial and can be seen from different points of view. In particular, there has been some phenomenologist analysis of this mistake (by J. Toussaint-Desanti). This error has been corrected with difficulty. Here is another way of stating the radical novelty of Luzin and Suslin's approach. But of course we don't pretend to go along with a religious explanation just as we do not believe in a phenomenological deconstruction.

In order to see what could be saved from Lebesgue's study, Suslin and later Luzin introduced a scheme, called Suslin's scheme, which can be represented symbolically by an infinite tree. It's basically a graph. Starting with zero, you have an infinite number of numbers: 0, 1, 2, and so on. It symbolizes the right way to "come close" to infinity, to approximate infinities with a finite construction of sets: a geometrical look at the old distinction between potential and actual infinity made by Aristotle.

An idea of the richness of the analytic subsets of the continuum, discovered by Luzin and his school, can be seen in a drawing made from the continuum of the plane, given by fractal pictures of the plane. The notations themselves lead naturally to considering non-denumerable cardinals. The class of analytic sets is rich and complicated. They satisfy the Continuum Hypothesis – that is, every uncountable analytic set is equinumerous with the set of all real numbers.

Of course, our example is not the only one of close connections between mathematical and philosophical thoughts. Interesting conclusions may be obtained by studying old and new examples, as in Pascal's "geometry of chance."

In another approach, Baruch Spinoza gives a very important role to infinity in his philosophy "*more geometrico*." And finally, one of the principal mathematicians of the recent period, Alexander Grothendieck, has provided penetrating analyses of the role of naming in the process of discovery. Remembering thirty years later his approach to a new geometry in 1958 with the notion of "*topoi*," he writes, "This vision was so obvious that I had not thought to give it a name, although it has always been my passion to name things that occur to me just as a first mean to apprehend them."

In the recent period, mathematics has developed new fields, with new symbols like the diagrams of arrows in categorical theories, thereby stimulating new intuitions that go well beyond what was known before. But there is still a mystery in the infinity and the continuum, as described beautifully by Gottfried Wilhelm Leibniz: "There are surely two labyrinths for the human mind: one is concerned with the making of continuum, the other with the nature of freedom, and they are born both from the same infinity." ■

Jean-Michel Kantor and Loren Graham, who spoke on Russian Religious Mystics and French Rationalists.

Allan Robinson (Harvard University) and Ronald Probstein (MIT)

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