Chapter 8

Mathematical Models

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I. INTRODUCTION

The application of mathematical models in water resources decision making has now become a fairly common and accepted practice. Many factors have contributed to this widespread use of models including: the enormous complexity of current water resource projects, the availability of computers, and more refined and proved systems analysis techniques. Despite this growth in the use of operational models and the frequent appearance of new models, there is little written for the layman on the philosophy, internal structure, and proper application of these models.

Mathematical modeling in water resources has become, and continues to become, such an esoteric and guarded art that only a small number of people in the country are qualified to interpret the results and inner workings of current models; and an even smaller group is able to develop new models, particularly those involving biological variables. These gurus of mathematical modeling are the experts who must be relied upon and consulted (often for considerable fees) by local, state, and federal agencies around the country who believe models will help solve their problems. This concentration of tools of analysis in the hands of so few is most unfortunate, for familiarity with these tools could be valuable to at least two large and important groups of people: (1) the decision makers themselves, and (2) concerned citizens who are directly affected by these decisions.

This essay is a modest attempt to explain mathematical modeling and the role of simulation, in terms the concerned layman can understand and apply. The Tocks Island Dam controversy is used as an illustrative example of the current use of mathematical modeling in water resources decision making. The role of a currently popular ecosystem simulation model in that controversy should serve as an interesting case history of the application of a major digital computer simulation model.

II. WATER RESOURCES MODELING

A. Evolution of Water Resources Modeling

Mathematical modeling of water resources dates back at least to 1925, when the classical Streeter-Phelps "oxygen sag" equation was first published.[1] It was a simple effort designed to simulate mathematically the dissolved oxygen concentration in a river downstream of a source of pollution. Although the mathematics used were unsophisticated and many factors affecting dissolved oxygen (DO) were not considered, the results were quite remarkable in how closely they simulated measured DO values. In fact, many present-day water resources studies[2,3] have used DO models principally based on this original oxygen sag equation, modified to handle additional DO factors.

From 1925 until the early sixties, most mathematical models were solved analytically, and environmental problems were studied largely by "hand calculations." This naturally limited the scope of the studies. However, starting in the early sixties, computers began to become larger, faster, and more accesible, and large mathematical modeling efforts became feasible. Problems that previously were too complex to solve by hand could now be approached. One of the first large scale water quality modeling efforts was the Delaware Estuary Comprehensive Study (DECS), which began in 1962 and ran for four years.[4] It focused on the Delaware DO distribution and has served as a basis for many subsequent modeling efforts.[5,6,7]

The use of mathematical models in water resources analysis and decision making can be of considerable importance to the ultimate disposition of a large water resources project. However, as with all evolving engineering tools of analysis, models have inherent constraints that determine the boundaries of analysis within which they may be properly applied. These constraints may take many shapes, but the principal concerns are the limitations associated with spatial dimensionality, time "dimensionality," and variables that measure the biological activity of the water. A firm understanding of a model's limitations is critical in assessing what questions a model can answer reliably and accurately. Misapplication of water resource models is commonplace, but so is underapplication: the failure to develop or use a model in a situation where the model would have been helpful.

B. Mathematical Models in General

In simple terms, mathematical modeling is a process that attempts to describe a dynamic, physical phenomenon by mathematical relationships which, when combined with accurate input data, imitate the real system. For example, if one considers a stagnant swimming pool, a differential equation could be written that equates the rate of temperature change in the pool to the net rate of evaporation, radiation, and convection occurring at the pool surface. After some simplifying assumptions, this equation could be solved, giving an algebraic expression that relates the time dependence of the temperature (the response) to the time dependence of some system disturbances, e.g., the ambient air temperature.

The advantages of using mathematical models to study a physical phenomenon are many. The given system may be studied as thoroughly as one wishes; i.e., any number of hypothetical alternative configurations may be probed, usually with a minimum of effort on the investigator's part. The investigator learns which variables are modeled easily and which need to be studied closely, leading to significant increases in overall project efficiency. In the swimming pool example, one could sit at the pool and measure the temperature for many months under diverse atmospheric conditions and plot such results on a multitude of graphs, each corresponding to different environmental conditions. This would take a great deal of time and the applicability of the results would be limited. The results would be contained in a mound of graphs; and because of the capriciousness of the weather, not all conditions would have been empirically studied. With a mathematical model and a computer, however, one could simulate an annual twelve-month weather cycle, through computer programmed meteorological relationships, and produce an output of temperature versus time for any period of the year and any specified environmental conditions. The savings in time and money, in addition to increased efficiency, are obvious.

In a water context, mathematical modeling is commonly used for water resources planning, management and decision making. Properly constructed models help our understanding of environmental behavior and broaden our information base, and thus they serve as invaluable tools in informed environmental decision making. Models are often used to predict the consequences of several viable alternative water resource plans that have a common data base and mathematical structure, but that differ in operational procedures.

C. Deterministic vs. Stochastic (Probabilistic) Models

The modeler who wishes to construct a mathematical model of a water quality variable, e.g., the temperature in a river, may use either of two approaches: deterministic or stochastic. (Occasionally a combination of both approaches is used, but this is a rare exception.) A deterministic model is one in which each variable and each parameter can be assigned a definite fixed number, or a series of fixed numbers, for any given set of conditions. In contrast, a stochastic model has uncertainty built into it: variables or parameters used to describe the input-output relationships and the constraints are not specified precisely.[8] Statistical techniques are used and results are expressed in language such as: "the value of dissolved oxygen will be 4 ± 1 mg/1 with 95 percent probability," meaning that in the long run the dissolved oxygen will be greater than 5 mg/1 or less than 3 mg/1 only 5 percent of the time.

Virtually all the large digital simulation models of water quality at present are deterministic in structure. Stochastic models are principally found in water quantity modeling, such as rainfall-runoff predictions and stream flow forecasting. The principles of deterministic modeling date back hundreds of years. By contrast, stochastic theory is relatively new, with most of the applications to water resources research appearing in the last fifteen years. For the foreseeable future, it appears that deterministic modeling will dominate water quality modeling. There are several reasons for this. Accurate and reliable stochastic models require extensive, precise and continuous records of data. Such data are rare; if a model of a future water resource, like a new lake, is desired, no such data *can* exist. The theoretical foundations for stochastic modeling are still being developed. And deterministic models have a track record: many water systems have been deterministically studied in a thorough and extensive manner, and several fundamental equations and relationships, such as the Streeter-Phelps equation, have been verified.

D. Modeling Variables and Transport Dynamics

In the simulation of water problems, the first choice is which water quality variables (responses or outputs) to model. The selection should reflect the questions being posed by the decision makers.

The most common variables used in water quality modeling are: dissolved oxygen (DO), biochemical oxygen demand (BOD, a measure of the strength of DO consuming wastes), temperature, and nutrients (typically phosphorous and nitrogen). Some other water quality indicators are turbidity, pH, conductivity, trace metal constituents, algal crop, total dissolved solids, bacteria concentration (particularly coliforms), and fish crop.

Complex and little understood chemical interactions and reactions, as well as gaps in the empirical data, make some variables more difficult to model mathematically than others. Thus the most pertinent parameter may not be modeled and environmental analysis may be based on how a secondary variable behaves. For example, most of the models of water quality in rivers have focused on predicting the concentration of dissolved oxygen along the length of the river. Two reasons for this focus are (1) we have had the most experience and success with this variable, and (2) an adequate DO concentration is a necessary condition for satisfactory water quality.

To be sure, in some rivers, the concentration of a particular toxic chemical would be much more pertinent to model than DO. However, because of a lack of understanding of how this chemical interacts, it may not be modeled, and instead the general health of the river may be measured only by its DO level. Unfortunately DO is not a sufficient condition for satisfactory river water quality; the river may be saturated with DO but unfit to bathe in due to high concentrations of toxic wastes.

Up until a few years ago, all the water quality variables used in models were "dead"—i.e., they did not describe organisms. The variables either acted like "good" chemicals, having fixed and well known kinetics or stoichiometric relationships (such as the DO used in satisfying a biochemical oxygen demand), or else were physical variables such as temperature. "Dead" variables are the most desirable to deal with (from an ease of modeling viewpoint) because they can be described by well known laws of physics and chemistry or by proved empirical relationships; and, in general, they obey these relationships regardless of the changing ambient conditions.

On the other hand, "live" variables, which have begun to appear in recent environmental modeling, are very difficult to simulate because we have not been able to define their behavior fully. One example of a live variable is phytoplankton, which has appeared in recent eutrophication models.[9] There are some empirical relationships that are being used to describe such live variables as phytoplankton, but often the description is crude, incomplete, or in partial error over a given time frame. The magnitude of the task of developing adequate models of live variables is described vividly in Daniel Goodman's essay, following. How much of that task can be accomplished as engineers and biologists begin to develop new working relationships remains to be seen.

All water bodies have characteristic hydrodynamic qualities. All reaches of rivers are marked by the presence of bulk advective flows, including estuaries, the portions of rivers influenced by tidal currents. In most rivers the primary advective flow is along the longitudinal axis, toward the ocean; this is the flow that is usually measured by a gauging station. In most estuaries two flows are occurring simultaneously: the fresh water current and the tidal current. The fresh water flow is continuous and is always toward the ocean; on the other hand, the tidal current is sometimes against the fresh water flow and sometimes with the flow, depending on whether flood tide or ebb tide conditions are operating. In rivers, when a pollutant is discharged, it is carried away downstream, never to be seen again. Estuaries, by contrast, transport the pollutant downstream on the ebb tide but return some of the waste on the flood tide; thus the pollutant moves back and forth in an oscillatory fashion, slowly making its way to the ocean because of the positive fresh water current. Lakes and reservoirs, in contrast, generally do not have a characteristic advective flow. Mixing and transport of material in such bodies of water is primarily caused by wind action, density effects, and concentration gradients.

An example of density influenced transport is the temperature stratification that occurs in all deep lakes. In general, a lake has a constant temperature in the early spring. As summer approaches, the upper layers (epilimnion) warm up, primarily through absorption of solar radiation. These layers, being at a higher temperature than the lower layers (hypolimnion), are less dense and tend to remain on top, producing a temperature stratification throughout the water body. In the fall the upper layers cool off, become more dense, and begin to sink. This causes a phenomenon known as the fall overturn; the entire lake may be mixed in the process. This can sometimes cause temporary water quality problems, when the lower quality water (low in DO and high in organic matter) in the hypolimnion mixes with the higher quality water in the epilimnion.

A mathematical model will trace the concentration of each of the water quality variables as it is advected either vertically or horizontally, as it diffuses (by molecular or turbulent transport) from regions of high concentrations to regions of low concentrations, and (if it is a "nonconservative" variable) as it undergoes chemical or biological reactions. The advective flows, diffusive transport, and chemical and biological reaction mechanisms are modeled by equations involving a few phenomenological parameters.

E. Space and Time

Space Dimensions. The present state of the art of large, operational, digital simulation models is rather elementary, in the sense that most models are one-dimensional in structure. This means that spatial variations of a given water quality parameter occur only in one direction, with homogeneity assumed for the remaining two dimensions. These models are therefore severely limited in the situations they represent accurately; however, they are often applied as a first approximation in a wide range of problems.

There are several reasons why substantial progress has not been made in developing two- and three-dimensional models. One reason is that real water resources problems are extremely complex, and our knowledge of internal processes (particularly hydrodynamic phenomena) is so incomplete that progressing beyond one dimension is thought by many to be unjustified, considering the many assumptions that would have to be made. Another reason is that two- and three-dimensional data bases are not available in sufficient quantity or quality, and thus model coefficients could not be calculated nor could such advanced models be fully verified. A third reason is the enormous demands on computer memory and time associated with two- and particularly three-dimensional numerical models, which greatly reduces their potential usefulness.

A final reason is that, in many cases, the questions being asked by decision makers are such that the approximate accuracy provided by a one-dimensional model is adequate for their purposes. For example, many rivers exhibit cross-sectional homogeneity (that is, pollutant concentrations vary only along the length of the river but neither across the river nor as a function of depth); in these cases, a one-dimensional model would be most suitable. One-dimensional models have also been demonstrated to be satisfactory for predicting the vertical variation of temperature in stratified lakes and reservoirs.[10,11]

Suppose a one-dimensional model is used to describe a nuclear power plant discharging hot water into a river. As represented in Figure 8-1, when a one-dimensional energy transport model is used to simulate the downstream temperature distribution, temperature variations in the Y and Z directions are assumed to be negligible, with temperature attenuation occurring only along the longitudinal or X axis (the downstream direction). This means that at any point, X_1,

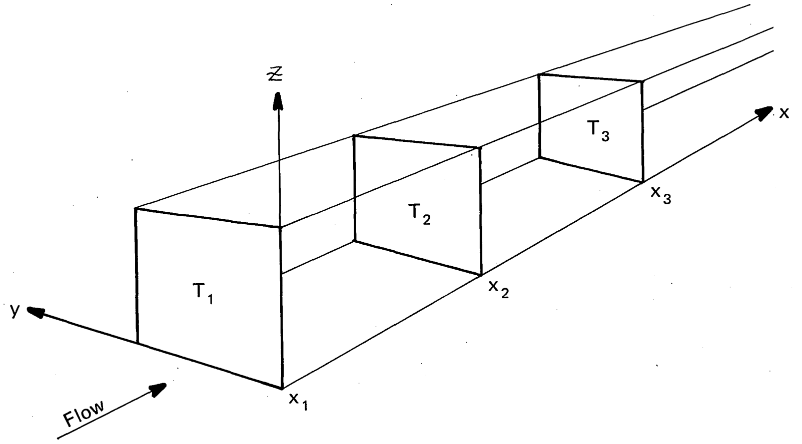

Figure 8-1. Temperature in a One-Dimensional Model

in the river, the temperatures at the top, middle, bottom, and sides of the river are the same, let us say T_1 (Fig. 8-1). At another point further downstream, X_2, the temperatures at the top, middle, bottom, and sides of the river are the same and equal to T_2; T_2 is numerically lower, however, than T_1. In effect, when a one-dimensional model is used, one is assuming that the modeled water quality variable, e.g., DO, BOD, or temperature, is homogeneous throughout the planes formed by the remaining two dimensions.

A two-dimensional model assumes variations occur in only two directions, with homogeneity in the third dimension. For example, let us again consider the modeling of temperature in a river. When hot water is discharged to a river it tends to remain in the upper layers due to density effects. This causes a vertical stratification with the hottest water on top and the coolest water at the bottom. As the water moves downstream, energy is lost to the ultimate sink: the atmosphere. As energy is lost, the temperature decreases in the longitudinal direction. Figure 8-2 illustrates how temperature varies in a two-dimensional model.

At X_1 in Figure 8-2, the surface temperature throughout the entire Y direction is T_1; however, the temperature decreases with depth, where T_1 is greater than T_1', which is greater than T_1''. At any fixed depth there is no lateral variation, i.e., the temperature is the same as one transverses the river from one bank to the other. At X_2 and X_3 a similar temperature structure is found, with the exception that the temperature is attenuated (by energy losses to the

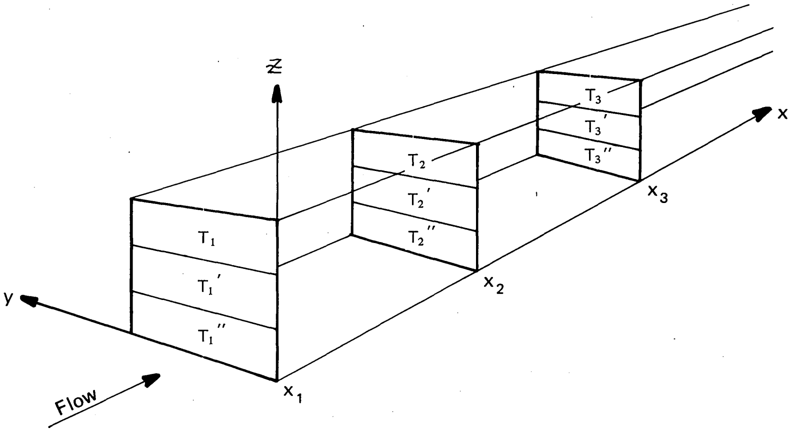

Figure 8–2. Temperature in a Two-Dimensional Model

atmosphere) at every depth, i.e., T_1 is greater than T_2, T_1' is greater than T_2' and so on.

A three-dimensional model is the most realistic possible, as variations in the longitudinal, vertical, *and* lateral directions are accounted for. In the example used previously, a three-dimensional model would predict temperature variations along the length of the river from the top of the river to the bottom, and from one bank to the other.

To take another example, consider the case of a single pipeline discharging secondary treated sewage into a river. After the waste enters the stream, it mixes vertically and laterally as it moves longitudinally downstream. Usually the depth of the stream is smaller than the width, and the waste will first become well mixed or homogeneous with respect to the depth dimension. Further downstream, the waste will continue to spread laterally until it contacts the banks, at which point it is laterally and vertically homogeneous. Below this point in the river (whose exact location depends, among other things, on the river dimensions and turbulence characteristics) the one-dimensional model may be applied to predict how the waste concentration will decay with distance. The model is not applicable above this point, although it is highly probable that one-dimensional models have been misused in this way. Because it may be a significant distance downstream before the one-dimensional model is appropriate, in many cases the waste (or temperature, in the case of thermal pollution) has had a chance to decay to a point where the concentration is essentially innocuous and there is not real point in modeling any additional decay.[a]

Most of the comprehensive, operational water resource models are one-dimensional. As environmental concerns increase and our questions become more complex, the demand for and use of more advanced, multidimensional models will also increase. There are many water resource problems that may be suitably approximated with a two-dimensional model. In rivers, because depths are relatively small, vertical mixing occurs rapidly and for many problems a two-dimensional model (lateral and longitudinal) is appropriate. For thermal pollution modeling, the vertical dimension cannot be neglected, because energy losses to the atmosphere result in vertical temperature stratification. For narrow rivers, however, a two-dimensional model is often suitable, in which variations in the lateral dimension are ignored. The development of comprehensive two-and three-dimensional water resources models is currently an area of active research, and significant advances can be expected over the next five years.

The Time Dimension. A steady state model is one in which there are no variations with time of the variables being modeled (e.g., DO) nor of any model parameters e.g., the river velocity or waste discharge rate. An unsteady state model, on the other hand, allows for variations with time. Unsteady state models are more complex and difficult to solve but they are evidently more realistic and comprehensive.

A steady state model may sometimes be used, even though in water resources problems very few processes are time invariant. Consider a model of the DO distribution in a river that has several industrial plants and combined sewer overflows discharging into it. During certain seasons, if the plants are operating at a constant rate of production and if there are no rain storms, the steady state DO models might be appropriate. However, if it suddenly rains and the combined sewers overflow into the river, or if the plants begin to operate irregularly, the steady state model cannot handle the resulting situation.[b]

[a] Y.L. Lau has demonstrated that one-dimensional temperature models are not very useful because by the time they are applicable, most of the serious temperature dissipation has taken place. He has presented an analytical two-dimensional model to improve on this situation.[12]

[b] Ackerman and Sawyer[13] were critical, and rightfully so, of the DECS DO model because it was steady state and could not properly account for the many time varying phenomena which occur in the Delaware estuary.

If a steady state model is used under unsteady state conditions, the results will certainly be in error, the magnitude of which will depend on the particular circumstances. The point for the nonexpert to remember is that a steady state model cannot account for any processes or effects that vary with time; if these time varying processes are a major input to the water resource system, then the results of using a steady state model should be vigorously scrutinized and used with caution.

III. MATHEMATICAL MODELS AND THE DECISION MAKER

Decision making in water resource problems involves choosing the best strategy from among several alternative courses of action. Because of the complex hydrological, ecological, and economic interactions and consequences, it is not an easy task, even for simple systems. However, if the decision maker has a mathematical model that simulates the behavior of the real water system, his task is made considerably easier. For, when this model is implemented on a high speed digital computer, an enormous number of alternative plans may be simulated and the consequences of each compared and contrasted. Models are invaluable tools for achieving intelligent planning, decision making and eventual management of a water resource. Without models, it is virtually impossible to arrive at informed environmental decisions for many of the comprehensive water resource problems facing federal and state agencies today.

The rules by which alternatives are compared must be specified, and this is usually not done by the modeler but by his client, a "decision maker." Models may be used to test alternatives but they do not choose which alternative is best for public policy. Modeling provides pertinent information and makes comparisons; the decision maker chooses the strategy. He must consider not only the results of his modeling effort but the implementability of each alternative. The most technically sound solution may not be the most politically sound. For example, if there were a number of industrial plants dumping wastes along a 50-mile stretch of a river, it is possible that the optimum solution, which achieves minimum pollution impact at a fixed total cleanup cost, will have 25 per cent of them using no waste treatment, 25 per cent using 60 per cent BOD removal and the remainder removing 95 per cent of their BOD. Although the solution is technically sound and economically optimal, it is probably impossible to implement, and other alternatives will have to be considered by the decision makers.

Although there is no question that mathematical models could be invaluable tools for the water resource decision maker or manager, they appear to me to be underused. There are many reasons for this unfortunate situation. Beginning in the early sixties, the term "systems analysis" became fashionable in all fields of engineering. Many of the early practitioners, excited about their new tools, developed boundless optimism for this sophisticated approach. In their exuberance, they convinced senior management personnel (who were trained long before the age of computers) to invest a great deal of time and money in their projects, with the promise that these new techniques would solve all the complex water resource problems imaginable. Not surprisingly, very few breakthroughs were made. In many cases substantial resources were wasted, and senior officials developed a hearty skepticism for any project or management tool with the word "model" or "systems" in it.

Many of the early modeling projects failed because the researchers were either unskilled in systems analysis techniques or did not have the water resources background to direct and apply these new methods properly; in other cases too much was promised or the models were misused. Over the last five years or so, slowly but steadily, more people have become skilled in both systems techniques and the fundamentals of water resource and environmental engineering problems. Concurrently, digital simulation models have been improved to the point where they are of considerable usefulness in a variety of water resource problems.

Modelers now are seeking to change the poor image early modeling efforts earned and to get models widely applied. They are beginning to take some of the mystery and "black box aura" out of modeling, writing user manuals so that the nonexpert can understand how the model was built and how to use it. The assumption is that when the decision makers are involved in the development, operation, and feedback improvement of the models, they will better appreciate the strengths and weaknesses of models, and much of the present skepticism will fade. If operational models for water resource decision making were located in widely accessible locations such as designated universities or federal agencies, where anyone could study and use them, this might help further. It is still true that certain important models are held by a handful of experts who charge for their services and whose findings cannot be independently checked.

An adequate awareness of the uncertainties in the output of mathematical models invariably disappears in the communication of results from modeler to decision maker. These uncertainties are not the least bit mysterious; they enter when our knowledge is either incomplete, unquantifiable, or not useful in an engineering sense.

Even when we are fortunate enough to know a great deal about a physical phenomenon, the mathematical formulation in its most complete form may be impossible to solve or perhaps too costly to solve in terms of computer time and memory. Therefore, in almost all modeling situations, assumptions and simplifications must be made that place bounds on the range of validity for the model.

The field of water resource management is replete with examples where numerical output of a model was treated as though the degree of uncertainty was zero. To try to flag this problem, a major ecosystem modeling study for the Office of Water Resources Research contained the following caveat at the end of the report: "The risk we run is that one may believe too strongly in the model *per se* and place too great an emphasis on absolute values to the detriment of the model's greatest capability—to compare choices."[14] Unfortunately, the sentiment expressed by this quote is too often not appreciated by those responsible for making decisions.

Uncertainty in water resource decision making is not pleasant to deal with. It would be nice if models produced results that could be depended upon with 100 percent certainty. But they never do. The fate of multimillion dollar projects may depend on how the nontechnically trained decision makers handle this uncertainty.

IV. MATHEMATICAL MODELING AND THE TOCKS ISLAND DAM CONTROVERSY

A. The McCormick Report: Eutrophication Is The Major Issue

The water resource decision making leading to the U.S. Corps of Engineers' Tocks Island Dam proposal occurred in the late 1950s, before large scale, operational mathematical models of water systems were available. Indeed, in those years very little consideration was given to the environmental and ecological consequences of damming a large river; had ecosystem models been available, they probably would not have been used.

In recent years there has been much opposition to the dam for a variety of reasons. One key issue, however, has been eutrophication. In 1971, McCormick and Associates,[15] under a contract from the Corps, predicted that accelerated cultural eutrophication would occur if the proposed impoundment were built.[c] They also concluded that if the river were dammed and no sewage effluents were allowed into the lake, i.e., current background conditions were

[c] Eutrophication means various things in various contexts, as described in the following essay. Here, the development of excessive plant growth of a noxious character is at issue.

perpetuated, the impoundment would undergo relatively slow natural eutrophication. The McCormick Report was based on very limited data and on some observations of other reservoirs (notably Cannonsville and Pepacton) in the region. Mathematical modeling of potential eutrophication was not done; indeed, the report concluded that "no existing model (September 1971) can describe or predict the complex interrelationships between physical, chemical, and biological parameters." Furthermore, McCormick and Associates felt that to develop such a model would take a "large commitment of funds" and a "truly national effort." Actually, at the time of their study, an attempt at such a model was in progress, but McCormick and Associates chose to dismiss it. Chen and Orlob, of the well known water resource consulting firm Water Resources Engineers (WRE), reported their initial ecologic modeling efforts to the Federal Water Quality Administration in 1968.[16] In October 1970, Chen published a paper[17] in a widely read technical journal that outlined an ecologic model applicable to rivers and impoundments; the model was "based on fundamental principles of biology, chemistry, and physics."

When the McCormick Report was received by the Corps, it undoubtedly caused considerable consternation. The report was very damaging in its eutrophication estimate; what's more, it seemed to leave the Corps with no recourse: without a model to attempt a second credible look at the question, the report and its damning conclusions appeared to lie unchallengable. However, fate was on the side of the Corps. Shortly after receiving the report, Dr. John Burnes, the chief of the environmental branch of the Philadelphia District of the Corps, attended an international conference at which Chen presented a paper on his ecologic modeling efforts and their application to impoundments. Burnes was quite surprised to learn of this model, since he had understood the McCormick Report to say that no such model existed. He also was quite delighted, for this model gave the Corps a viable alternative approach to eutrophication assessment; the worst that could possibly happen would be that the model would confirm the predictions of cultural eutrophication problems, and there was always the chance that results would be predicted in support of the Corps' position.

The Corps contracted with WRE to adapt their general lake ecologic model to the Tocks Island case, and in February 1973 WRE submitted its report to the Corps.[18] WRE concluded that Tocks Island Lake (the report is careful rarely to mention the word *dam*) "appears capable of supporting a well balanced ecosystem without undesirable quality changes as long as careful attention is given to the nature and magnitude of nutrient loading." It included in what it considered "careful attention," the application of advanced waste treatment, including 95 per cent phosphorous reduction relative to secondary treated effluent.

B. LAKECO: An Ecosystem Simulation Model

WRE's lake eutrophication model, known as LAKECO, is perhaps the best state of the art, operational ecologic simulation model for reservoirs in the world today. It has been adopted by the U.S. Corps of Engineers and is presently available to all Corps offices through the Corps' Hydrologic Engineering Center (HEC) in Davis, California. Since WRE's Tocks Island ecologic simulation study concludes that eutrophication will not be a major problem, provided advanced waste treatment is practiced, it would appear to nullify a major objection to building the dam. But the report has not had this effect. There are many private doubts at the Corps and the DRBC about LAKECO's predictive capabilities. These doubts have evidently permeated the decision making processes in several quarters, and the eutrophication issue still remains very much alive.

LAKECO has been the major mathematical model involved in Tocks Island decision making to date, and WRE's conclusions are favorable to dam proponents. Many feel secure, perhaps even smug, in their position, because they are backed by a sophisticated technological tool and recognized modeling experts, and because opponents of the dam have no comparable technological support. Since LAKECO not only is important to the Tocks Island controversy but no doubt will also be used in eutrophication disputes and decision making throughout the country (HEC plans to distribute user manuals to Corps offices), it is important to understand LAKECO's internal structure, capabilities, limitations, and credibility. Such understanding should help to put LAKECO in a reasonable perspective and aid in its proper application and interpretation.

The History of LAKECO. Water Resources Engineers is over ten years old, making it one of the oldest consulting firms of its type in the United States. Its first major effort was a series of water quality models of San Francisco Bay and Delta. Then, in 1967, the firm began to develop and refine a one-dimensional temperature model for streams and reservoirs, supported first by the State of California's Department of Fish and Game, then by the Federal Water Pollution Control Administration (now the Environmental Protection Agency), the United States Army Corps of Engineers, and the State of Washington's Pollution Control Commission. The WRE temperature model, which forms the basis for LAKECO, has had some difficulties in application and is regarded with caution by workers in the field.[d]

In 1968, WRE submitted a report to the Southwest Region of EPA entitled: "A Proposed Ecologic Model for Eutrophying Environment." This was the initial phase in the development of LAKECO. In 1970, Chen published a paper outlining WRE's ecologic simulation model for rivers and reservoirs, and preliminary tests of the model were reported for a one-dimensional, segmented river. In July 1970, the Office of Water Resources Research (OWRR) gave a two-year contract to WRE to develop a general purpose model for the simulation of aquatic ecosystem behavior. In December 1972, WRE submitted their final report to OWRR. As they had done several times previously, WRE refused to provide a program listing for public use, despite being funded by public funds.

The model became available to the public only as a result of the efforts of the Corps of Engineers, whose contract required WRE to supply the model as well as to advise and train the Corps in operating it. It is now a major model in the Corps' arsenal of water quality simulation models. I expect it to be used frequently, for the eutrophication issue has been such a successful weapon for environmentalists who oppose the Tocks Island Dam that it will no doubt be cited by other environmental groups who oppose dam projects throughout the country.

At first acquaintance, LAKECO appears formidable. It simultaneously simulates 22 variables including: DO, BOD, pH, temperature, phosphorous, nitrogen (3 forms), algae (3 types), fish (3 types), zooplankton, and benthic animals. In a typical simulation each variable has its own distinct differential equation, which is also subscripted with a designation of the specific slice in the vertical direction. Thus, when ten variables are being simulated and a reservoir 150 meters deep is divided into two-meter slices, 750 differential equations must be considered. In addition there are hydrodynamic equations that describe how flows move in the system. Many of these equations are coupled, because many of the concentrations affect one another. In spite of all this complexity, the equations can in fact be handled with no great difficulty on a modern computer. The most significant problems associated with LAKECO lie elsewhere: in its hydrodynamics, considered below, and in its biology, considered in the next essay.

[d] Specifically, I am familiar with one experience where Battelle Northwest (Richland, Washington) found they had to rewrite the program completely.

Hydrodynamic Simulation in LAKECO. The hydrodynamic scheme in LAKECO is rather simple in concept. The underlying philosophy is that water seeks its own density layer. Water flowing into the reservoir has a certain density based primarily on its temperature: it enters the layer in the lake that has the same temperature, and then *instantaneously* mixes through the volume of that layer. Since water is assumed incompressible, the inflowing water to a layer displaces water from that layer, which causes a net advective flow along the vertical axis. Outflow from the reservoir may occur from one or more of the outlets at various depths, depending on the particular discharge pattern desired. It is assumed that the outlet may be modeled as a slit having the same width as the reservoir dam at that level. Outflowing water comes from the layer at that elevation and from layers just above and below, in accordance with density based criteria. If the flow into the lake is not equal to the flow out, this net advective vertical flow results in a change in the surface level of the lake.

LAKECO, as a one-dimensional model, is composed of vertically stacked, continuously stirred tank reactors (CSTR's), in each of which physical, chemical, and biological phenomena can occur. The concentration of abiotic and biotic constituents and the temperature in each tank may change with time but never with spatial position within the tank. A tank typically represents a slice of reservoir two meters thick; the thinner the tanks, the more accurate the model, but the higher the cost in computer time. Since the tanks are vertically arranged, LAKECO is strictly applicable to a longitudinally and laterally well mixed reservoir where variations in concentration are significant only along the vertical direction. Figure 8-3 illustrates the spatial segmentation required by the model.

However, the cost of such simplification is a loss in accuracy and realism. By definition, the flow entering any CSTR instantaneously mixes with the entire volume of the tank. In the case of Tocks Island Lake, this means that if one pound of phosphorous is dumped in at the upstream end of the lake, it instantaneously mixes over the entire length of the lake (a distance of 37 miles), over the entire width, and over the depth of the stirred tank slice. Such incredible mixing would predict a tremendous and misleading dilution effect on any nutrients entering the lake. One doesn't have to be a mathematical modeler to be justifiably skeptical of the results predicted by a model that instantaneously mixes nutrients throughout a volume of water 37 miles long, up to 3,000 feet wide and approximately six feet deep. Thirty-seven miles of water can neutralize the effect of almost any

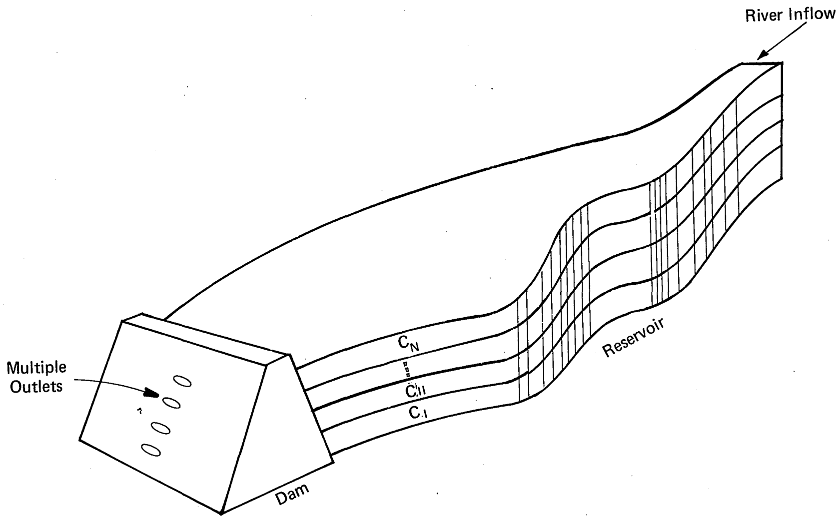

Figure 8-3. Horizontal Reservoir Slices used in LAKECO.

contaminant. Such dilution power has long been the dream of many an industrial water manager!

WRE justifies its one-dimensional model by the following interesting quote:[18]

> Orlob and Selna[e] have shown by comparison of simulated and observed behavior that the one-dimensional representation is satisfactory for reservoirs and lakes that exhibit pronounced annual cycles of thermal stratification.

This statement is patently misleading, for while Orlob and Selna (as well as others) have shown that for many reservoirs a one-dimensional temperature representation is adequate, the WRE statement implies that for stratified reservoirs, *all* water quality variables (notice they do not distinguish any particular water quality variable) may be adequately represented by one-dimensional models.

Because temperature varies directly with density, and because perturbations of temperature by sun and wind are driven primarily in the vertical direction, it is not surprising that in many reservoirs, temperature varies almost exclusively in the vertical direction. However, in the case of such water quality variables as nitrogen, phosphorous, and BOD, perturbations occur along the longitudinal axis,

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[e] The work under discussion by Orlob and Selna is our note 10.

i.e., in the direction of the river flow, and therefore one would expect concentration variations to occur at least along this axis.

In any dammed-up river, the resulting lake tends to be elongated, with water quality variations occuring along the length of the lake. For example, because of phosphorous' tenacity in attaching itself to soil particles, one would expect the upstream end of a reservoir to have higher concentrations of phosphorous than the area near the dam (the heavier soil particles would tend to settle out at the upstream end). Therefore one might expect to find local algal blooms at the upstream end whose effect would gradually move towards the dam.

LAKECO cannot predict this local, longitudinal effect. It instantaneously mixes the phosphorous and soil particles throughout the volume of the layer nearest the inflow density, and thus dilutes the effect of phosphorous to an inconsequential level. Conversely, in the case of a discharge of sewage effluent at a point near the downstream end of the reservoir, the effluent might actually pass rapidly through the outlet with negligible effect on the lake. However, LAKECO would instantaneously mix this discharge throughout the volume of its discharge layer, thus making nutrients available to every chunk of fluid in the layer volume, and might conceivably predict an algal bloom when in reality there could be none.

In summary, LAKECO is inherently capable of simulating only longitudinally and laterally well mixed reservoirs and lakes. It is highly limited in its applicability to long reservoirs (such as the one that would be formed by the Tocks Island Dam, or a dam on almost any river), since significant concentration variations will generally occur along the length of the reservoir; the errors introduced increase with the length of the reservoir. In the case of a lake as long as Tocks Island Lake, the errors would be expected to be of such magnitude as to preclude any definitive analysis by LAKECO, and at least a two-dimensional (longitudinal and vertical) model would appear to be required. Unfortunately, two-dimensional, comprehensive eutrophication models are not presently available.[f]

V. CONCLUSION

LAKECO has certainly contributed to the advancement of the state of the art of a field still in its infancy, and for this WRE will be remembered. Before LAKECO, "advanced" ecosystem models generally assumed complete mixing and contained no hydrodynamic

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[f] This section benefitted from several analyses of LAKECO done by Douglas Zaeh, a Princeton undergraduate.

subroutines. As most of these early studies dealt with small, shallow ocean bays or small ponds, the assumption of complete mixing could be lived with. LAKECO now allows us to extend these early efforts to deep water bodies where vertical concentration variations may be modeled by a series of stacked stirred tanks. But for many applications LAKECO and its next several successors are likely to be inadequate, and water resources decision makers who consult these models to broaden their information base must be kept aware of their shortcomings and questionable assumptions.

In the particular case of Tocks Island, the application of LAKECO has clearly not resulted in a clarification of the eutrophication issue; indeed, it may have contributed to even more clouding of the question. Because of the length of the Tocks reservoir, important longitudinal concentration variations are certain to occur, and LAKECO is completely incapable of predicting these changes. In addition to inherent hydrodynamic limitations, the 365-day Tocks simulation was based on only 24 daily grab samples of nitrate data and 20 daily grab samples of phosphate data. These data were massaged by a special "data generator" program, which precedes LAKECO, into 365 days of nitrate and phosphorous data.

The McCormick Report was most emphatic on the lack of input water quality data, and its recommendation that more data be collected was not carried out in time for the WRE simulation. Hence, even if the model were perfect, the data base used is so pathetically small (and the numerical values in some cases so questionable) that accurate model predictions are most certainly precluded. In addition to the lack of water quality data, the weather data used to model the reservoir temperature also is somewhat limited, and in the case of solar radiation was taken from New York City, some 60 miles away.

In summary, the very poor data base, combined with the inherent one-dimensional character of LAKECO, makes the WRE eutrophication conclusions highly suspect. It is recommended that these LAKECO derived conclusions be excluded completely in future Tocks Island decision making.

NOTES

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